Module 2: Fundamentals of AC Circuits

Module Description:

This module provides a comprehensive extension of circuit analysis to alternating current (AC) systems. We will begin by thoroughly understanding the fundamental characteristics of sinusoidal waveforms, including their mathematical representation and key parameters. A cornerstone of AC circuit analysis, phasor representation, will be introduced as a powerful tool for simplifying complex calculations involving phase differences. You will gain proficiency in analyzing single-phase AC circuits containing individual resistors, inductors, and capacitors, as well as their intricate series and parallel combinations, by leveraging the concept of complex impedance. The module then transitions to the critical topic of power in AC circuits, distinguishing between instantaneous, real, reactive, and apparent power, and explaining the significance of the power factor and power triangle. We will delve into the unique phenomenon of resonance in RLC circuits, exploring both series and parallel configurations and their associated parameters like quality factor and bandwidth. Finally, the module concludes with an in-depth look at three-phase balanced systems, covering their advantages, common connection types (Star and Delta), and power calculations, which are essential for understanding modern electrical power distribution.

Learning Objectives:

Upon successful completion of this module, you will be able to:

- **Comprehensively describe and mathematically represent sinusoidal waveforms, including their frequency, period, amplitude, and phase angle.**
- **Accurately calculate peak, RMS, and average values for various AC quantities and understand their practical significance.**
- **Perform detailed analysis of single-phase AC circuits containing combinations of resistive, inductive, and capacitive components in both series and parallel configurations, utilizing complex impedance and admittance concepts.**
- **Define, differentiate, and precisely calculate instantaneous, real (average), reactive, and apparent power in AC circuits, along with the crucial power factor and its representation in the power triangle.**
- **Thoroughly explain the phenomenon of resonance in RLC circuits, identify resonant frequencies, and calculate associated parameters like quality factor and bandwidth for both series and parallel resonant circuits.**
- **Analyze and solve problems related to three-phase balanced circuits, understanding the relationships between line and phase voltages/currents for Star (Wye) and Delta connections, and calculating power in such systems.**

Topics:

1. Sinusoidal Waveforms: The Foundation of AC

Alternating current (AC) is defined by its characteristic sinusoidal variation over time, making it fundamentally different from direct current (DC). This section lays the groundwork for understanding AC phenomena.

- **Generation of Sinusoidal Waveforms:**
  - **Sinusoidal AC voltage (and consequently current) is primarily generated by electromagnetic induction. When a coil rotates uniformly within a uniform magnetic field (as in an alternator or generator), the rate of change of magnetic flux linkage through the coil varies sinusoidally. This sinusoidal rate of change induces a sinusoidal electromotive force (EMF), which drives the current in a closed circuit.**
  - **Mathematically, if a conductor of length 'l' moves with velocity 'v' perpendicular to a magnetic field 'B', the induced voltage is $e=Blv$. In a rotating coil, the component of velocity perpendicular to the field varies sinusoidally, leading to a sinusoidal induced voltage.**
  - **Electronic oscillators (e.g., LC oscillators, RC phase-shift oscillators) can also generate sinusoidal waveforms by utilizing the energy storage and dissipation characteristics of reactive components.**
- **Key Parameters of a Sinusoidal Waveform: A general sinusoidal voltage (or current) can be expressed as a function of time: $v(t)=V_m\sin(\omega t+\phi)$ or $v(t)=V_m\cos(\omega t+\phi)$ Where:**
  - **$v(t)$: Instantaneous voltage at time 't'.**
  - **V_m: Peak (or maximum) value of the voltage. This is the amplitude of the sine wave.**
  - **ω: Angular frequency in radians per second (rad/s). It describes how fast the sine wave oscillates.**
    - **Formula: $\omega=2\pi f$**
    - **The term ωt represents the angular displacement in radians at time t.**
  - **t: Time in seconds.**
  - **ϕ: Phase angle (or phase shift) in radians or degrees. It indicates the position of the waveform relative to a reference at $t=0$. A positive ϕ means the waveform is shifted to the left (leading), and a negative ϕ means it's shifted to the right (lagging).**
- **Frequency (f):**
  - **Definition: The number of complete cycles of the waveform that occur in one second. It quantifies how frequently the waveform repeats.**
  - **Units: Hertz (Hz). One Hertz means one cycle per second.**
  - **Formula: $f=1/T$ (where T is the period).**
- **Period (T):**
  - **Definition: The time required for one complete cycle of the waveform to occur. It is the reciprocal of frequency.**
  - **Units: Seconds (s).**
  - **Formula: $T=1/f$**
- **Amplitude (V_m or I_m):**

- Definition: The maximum instantaneous value attained by the voltage (V_m) or current (I_m) during a cycle. It's the height of the waveform from its center line to its peak.
- Phase Angle (ϕ or θ):
    - Definition: The angular displacement of a sinusoidal waveform from a reference point at $t=0$. When comparing two waveforms of the same frequency, their phase difference indicates whether one waveform "leads" (occurs earlier) or "lags" (occurs later) the other.
    - If $v_1(t)=V_{m1}\sin(\omega t+\phi_1)$ and $v_2(t)=V_{m2}\sin(\omega t+\phi_2)$:
        - If $\phi_1>\phi_2$, $v_1(t)$ leads $v_2(t)$ by $(\phi_1-\phi_2)$ degrees/radians.
        - If $\phi_1<\phi_2$, $v_1(t)$ lags $v_2(t)$ by $(\phi_2-\phi_1)$ degrees/radians.
        - If $\phi_1=\phi_2$, they are in phase.
        - If $|\phi_1-\phi_2|=180^\circ$ (or π radians), they are out of phase (or anti-phase).
- Numerical Example 1.1: An AC voltage waveform is described by the equation $v(t)=325\sin(377t+60^\circ)$ V. Determine its amplitude, angular frequency, frequency, period, and phase angle.
    - Amplitude (V_m): By direct comparison with $V_m\sin(\omega t+\phi)$, $V_m=325$ V.
    - Angular Frequency (ω): $\omega=377$ rad/s.
    - Frequency (f): $f=\omega/(2\pi)=377/(2\pi)\approx 60$ Hz.
    - Period (T): $T=1/f=1/60\approx 0.01667$ s or 16.67 ms.
    - Phase Angle (ϕ): $\phi=60^\circ$ (leading). This means the waveform starts 60° earlier than a reference sine wave at $t=0$.

2. AC Quantities: Effective Values

Since AC voltages and currents are constantly changing, we need specific metrics to quantify their "effective" or "equivalent" values for practical circuit analysis and power calculations.

- Peak Value (V_m or I_m):
    - Definition: The maximum instantaneous value of the waveform reached during a cycle. It is the amplitude.
- RMS Value (Root Mean Square) (V_{RMS} or I_{RMS}):
    - Definition: The RMS value of an AC quantity is the equivalent DC value that would produce the same amount of heat (or power dissipation) in a given purely resistive circuit. It is the most commonly used and cited value for AC voltages and currents in power systems and specifications (e.g., "230 V AC" refers to the RMS value).
    - Derivation (for any periodic waveform): The RMS value is calculated by taking the square root of the mean (average) of the squares of the instantaneous values over one complete cycle. $V_{RMS}=\sqrt{\frac{1}{T}\int_0^T[v(t)]^2dt}$
    - Formula for Pure Sinusoidal Waveform: For a pure sine wave, the relationship between peak and RMS values is fixed: $V_{RMS}=V_m/\sqrt{2}\approx 0.707V_m$ $I_{RMS}=I_m/\sqrt{2}\approx 0.707I_m$
- Average Value (V_{avg} or I_{avg}):
    - Definition: The average value of a symmetrical sinusoidal waveform over a complete cycle is zero, as the positive half-cycle cancels out the

negative half-cycle. Therefore, the average value is typically considered over a half-cycle (usually the positive half-cycle).

- Derivation (for any periodic waveform): The average value is calculated by taking the average of the instantaneous values over one half-cycle. $Vavg=T/21\int 0T/2v(t)dt$
- Formula for Half-Cycle of Pure Sinusoidal Waveform: $Vavg=(2/\pi)Vm\approx 0.637Vm$ $Iavg=(2/\pi)Im\approx 0.637Im$

- Form Factor and Peak Factor:
  - Form Factor (FF): Ratio of RMS value to Average value. For a sine wave, $FF=(Vm/2)/(2Vm/\pi)=\pi/(22)\approx 1.11$.
  - Peak Factor (Crest Factor) (PFk): Ratio of Peak value to RMS value. For a sine wave, $PFk=Vm/(Vm/2)=2\approx 1.414$.
- Numerical Example 2.1: A sinusoidal AC current has an RMS value of 10 A. Calculate its peak value and average value (over a half-cycle).
  - Peak Value (Im): $IRMS=Im/2\Rightarrow Im=IRMS\times 2=10\times 2\approx 14.14$ A
  - Average Value (Iavg): $Iavg=(2/\pi)Im=(2/\pi)\times 14.14\approx 0.637\times 14.14\approx 9.01$ A

3. Phasor Representation: Simplifying AC Analysis

Phasors provide a powerful graphical and mathematical tool to represent sinusoidal quantities, making AC circuit analysis as straightforward as DC circuit analysis by converting time-domain functions into static complex numbers.

- Concept of a Phasor:
  - A phasor is a rotating vector that graphically represents a sinusoidal quantity (voltage or current).
  - Its length represents the amplitude (typically the RMS value, though peak value can also be used, consistency is key).
  - Its angle with respect to a reference axis (usually the positive real axis) represents the phase angle of the waveform at time $t=0$.
  - Phasors are assumed to rotate counter-clockwise at the angular frequency ω. By "freezing" them at a specific time (usually $t=0$), we can represent the relative phase relationships between different quantities.
  - Example: A voltage $v(t)=Vmsin(\omega t+\phi)$ can be represented as a phasor $V=(VRMS)\angle\phi$, where $VRMS=Vm/2$.
- Complex Plane and Complex Numbers:
  - Phasors are mathematically represented as complex numbers in the complex plane. A complex number Z can be expressed in:
    - Rectangular Form: $Z=x+jy$, where x is the real part and y is the imaginary part. j is the imaginary unit, where $j=-1$.
    - Polar Form: $Z=|Z|\angle\theta$, where $|Z|$ is the magnitude (modulus) and θ is the angle (argument).

- **Conversion between Forms:**
  - **From Rectangular to Polar: $|Z|=\sqrt{x^2+y^2}$ $\theta=\arctan(y/x)$ (paying attention to the quadrant of x and y)**
  - **From Polar to Rectangular: $x=|Z|\cos\theta$ $y=|Z|\sin\theta$**

- **Complex Impedance (Z): The AC Equivalent of Resistance**
  - **In AC circuits, the total opposition to current flow is called impedance, denoted by Z. Impedance is a complex number that accounts for both energy dissipation (resistance) and energy storage (reactance).**
  - **Ohm's Law for AC Circuits (Phasor Form): $V=IZ$, $I=V/Z$, $Z=V/I$. Here V and I are voltage and current phasors, and Z is the complex impedance.**
  - **Impedance of a Resistor (Z_R):**
    - **A resistor dissipates energy but does not store it. In a purely resistive circuit, voltage and current are always in phase.**
    - **Formula: $Z_R=R\angle 0^\circ=R+j0$ (Ohm's). The impedance is purely real.**
  - **Impedance of an Inductor (Z_L):**
    - **An inductor stores energy in its magnetic field. In a purely inductive circuit, the current lags the voltage by 90°.**
    - **Inductive Reactance (X_L): The opposition offered by an inductor to the change in current.**
      - **Formula: $X_L=\omega L=2\pi fL$ (Ohms).**
    - **Complex Impedance: $Z_L=jX_L=X_L\angle 90^\circ$. The impedance is purely imaginary and positive.**
  - **Impedance of a Capacitor (Z_C):**
    - **A capacitor stores energy in its electric field. In a purely capacitive circuit, the current leads the voltage by 90°.**
    - **Capacitive Reactance (X_C): The opposition offered by a capacitor to the change in voltage.**
      - **Formula: $X_C=1/(\omega C)=1/(2\pi fC)$ (Ohms).**
    - **Complex Impedance: $Z_C=-jX_C=X_C\angle -90^\circ$. The impedance is purely imaginary and negative.**
  - **General Complex Impedance (Z):**
    - **For a circuit containing a combination of R, L, and C, the total impedance is represented in rectangular form as: $Z=R+j(X_L-X_C)$ Where R is the net resistance and (X_L-X_C) is the net reactance.**
    - **In polar form: $Z=|Z|\angle\theta$**
      - **Magnitude of Impedance: $|Z|=\sqrt{R^2+(X_L-X_C)^2}$**
      - **Impedance Angle: $\theta=\arctan((X_L-X_C)/R)$. This angle represents the phase difference between the total voltage across the impedance and the total current flowing through it. If θ is positive, the voltage leads the current (inductive circuit); if negative, the voltage lags the current (capacitive circuit).**

- **Numerical Example 3.1: A series circuit consists of a 20Ω resistor, a 0.1 H inductor, and a 100μF capacitor, connected to a 230 V, 50 Hz AC supply. Calculate the total impedance of the circuit.**
  - **Resistance (R): $R=20\Omega$. So $Z_R=20\angle 0^\circ=20+j0\ \Omega$.**
  - **Inductive Reactance (X_L): $\omega=2\pi f=2\pi\times 50=314.16$ rad/s. $X_L=\omega L=314.16\times 0.1=31.416\ \Omega$. So $Z_L=31.416\angle 90^\circ=j31.416\ \Omega$.**

- **Capacitive Reactance (XC):** $X_C = 1/(\omega C) = 1/(314.16 \times 100 \times 10^{-6}) = 1/(0.031416) \approx 31.83\ \Omega$. So $Z_C = 31.83\angle -90^\circ = -j31.83\ \Omega$.
- **Total Impedance (Ztotal):** $Z_{total} = Z_R + Z_L + Z_C = (20 + j0) + (0 + j31.416) + (0 - j31.83)$ $Z_{total} = 20 + j(31.416 - 31.83) = 20 - j0.414\ \Omega$ (Rectangular form)
- **Convert to Polar Form:** $|Z_{total}| = 20^2 + (-0.414)^2 = 400 + 0.1714 \approx 400.1714 \approx 20.004\ \Omega$ $\theta = \arctan(-0.414/20) = \arctan(-0.0207) \approx -1.186^\circ$ $Z_{total} = 20.004\angle -1.186^\circ\ \Omega$. This indicates a slightly capacitive circuit overall.

4. AC Circuit Analysis: Applying Phasors

With phasors and complex impedance, Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL) can be applied to AC circuits in the same way they are used for DC circuits, significantly simplifying calculations.

- **Individual Components in AC Circuits:**
  - **Purely Resistive Circuit:**
    - **Phase Relationship: Current and voltage are in phase ($\phi = 0^\circ$).**
    - **Ohm's Law: $V = IR$ or $V = IR$ (magnitudes).**
  - **Purely Inductive Circuit:**
    - **Phase Relationship: Current lags voltage by 90° ($\phi = -90^\circ$).**
    - **Ohm's Law: $V = I(jX_L)$. In magnitude, $V = IX_L$.**
  - **Purely Capacitive Circuit:**
    - **Phase Relationship: Current leads voltage by 90° ($\phi = +90^\circ$).**
    - **Ohm's Law: $V = I(-jX_C)$. In magnitude, $V = IX_C$.**
- **Series Combinations (RL, RC, RLC Series):**
  - **Characteristic: The current is the same through all series components. The total voltage is the phasor sum of individual component voltages.**
  - **Total Impedance: The total impedance of series-connected components is the phasor sum of their individual impedances: $Z_{total} = Z_1 + Z_2 + ... + Z_n$ For RLC series: $Z_{total} = R + jX_L - jX_C = R + j(X_L - X_C)$**
  - **Current Calculation: Using Ohm's Law for AC: $I = V_{source}/Z_{total}$**
  - **Voltage Across Components:**
    - **Voltage across Resistor: $V_R = IZ_R = IR$**
    - **Voltage across Inductor: $V_L = IZ_L = I(jX_L)$**
    - **Voltage across Capacitor: $V_C = IZ_C = I(-jX_C)$**
  - **Phasor Diagram for Series RLC:**
    - **Choose the current phasor (I) as the reference (horizontal).**
    - **V_R is in phase with I.**
    - **V_L leads I by 90°.**
    - **V_C lags I by 90°.**
    - **The source voltage V_{source} is the phasor sum of V_R, V_L, and V_C.**
- **Numerical Example 4.1 (RL Series Circuit): A 15Ω resistor is in series with an inductor with $X_L = 20\Omega$. The series combination is connected to a 120 V, 60 Hz AC supply. Calculate the total impedance, total current, voltage across the resistor, and voltage across the inductor.**

- Total Impedance (Ztotal): $Z_R=15\angle 0^\circ=15+j0\ \Omega$ $Z_L=20\angle 90^\circ=0+j20\ \Omega$ $Z_{total}=Z_R+Z_L=(15+j0)+(0+j20)=15+j20\ \Omega$
    - In polar form: $|Z_{total}|=\sqrt{15^2+20^2}=\sqrt{225+400}=\sqrt{625}=25\ \Omega$
    - $\theta=\arctan(20/15)=\arctan(1.333)\approx 53.13^\circ$
    - So, $Z_{total}=25\angle 53.13^\circ\ \Omega$.
- Total Current (I): Assume the supply voltage is the reference: $V_{source}=120\angle 0^\circ$ V. $I=V_{source}/Z_{total}=(120\angle 0^\circ)/(25\angle 53.13^\circ)=(120/25)\angle(0^\circ-53.13^\circ)=4.8\angle -53.13^\circ$ A. (The current lags the voltage by 53.13°, as expected for an inductive circuit).
- Voltage across Resistor (VR): $V_R=I\times Z_R=(4.8\angle -53.13^\circ)\times(15\angle 0^\circ)=(4.8\times 15)\angle(-53.13^\circ+0^\circ)=72\angle -53.13^\circ$ V.
- Voltage across Inductor (VL): $V_L=I\times Z_L=(4.8\angle -53.13^\circ)\times(20\angle 90^\circ)=(4.8\times 20)\angle(-53.13^\circ+90^\circ)=96\angle 36.87^\circ$ V.
- Verification (KVL): $V_R+V_L=(72\cos(-53.13^\circ)+j72\sin(-53.13^\circ))+(96\cos(36.87^\circ)+j96\sin(36.87^\circ))=(43.2-j57.6)+(76.8+j57.6)=120+j0=120\angle 0^\circ$ V (matches source voltage).

- Parallel Combinations (RL, RC, RLC Parallel):
  - Characteristic: The voltage is the same across all parallel branches. The total current is the phasor sum of the individual branch currents.
  - Admittance (Y): For parallel circuits, it's often more convenient to work with admittance, which is the reciprocal of impedance ($Y=1/Z$). Admittance is also a complex number: $Y=G+jB$ Where:
    - G: Conductance (reciprocal of resistance, $G=1/R$). Measured in Siemens (S).
    - B: Susceptance (reciprocal of reactance, $B=1/X$). Measured in Siemens (S).
      - Inductive Susceptance: For $Z_L=jX_L$, $Y_L=1/(jX_L)=-j(1/X_L)=-jB_L$.
      - Capacitive Susceptance: For $Z_C=-jX_C$, $Y_C=1/(-jX_C)=j(1/X_C)=jB_C$.
  - Total Admittance: The total admittance of parallel-connected components is the phasor sum of their individual admittances: $Y_{total}=Y_1+Y_2+...+Y_n$ For RLC parallel: $Y_{total}=G+jB_C-jB_L=G+j(B_C-B_L)$
  - Total Current Calculation: Using Ohm's Law for AC: $I_{total}=V_{source}Y_{total}$
  - Current Through Branches:
    - Current through Resistor: $I_R=VY_R=V(1/R)$
    - Current through Inductor: $I_L=VY_L=V(-jB_L)$
    - Current through Capacitor: $I_C=VY_C=V(jB_C)$
  - Phasor Diagram for Parallel RLC:
    - Choose the source voltage phasor (V_{source}) as the reference (horizontal).
    - I_R is in phase with V_{source}.
    - I_L lags V_{source} by 90°.
    - I_C leads V_{source} by 90°.
    - The total current I_{total} is the phasor sum of I_R, I_L, and I_C.

- **Numerical Example 4.2 (RL Parallel Circuit): A 50Ω resistor is in parallel with a 0.2 H inductor. The parallel combination is connected to a 100 V, 50 Hz AC supply. Calculate the total current drawn from the supply.**
  - **Angular Frequency (ω): $\omega=2\pi f=2\pi\times50=314.16$ rad/s.**
  - **Inductive Reactance (XL): $X_L=\omega L=314.16\times0.2=62.83\ \Omega$.**
  - **Admittance of Resistor (YR): $Y_R=1/R=1/50=0.02$ S. In polar form: $0.02\angle0^\circ$ S.**
  - **Admittance of Inductor (YL): $Y_L=1/Z_L=1/(jX_L)=1/(j62.83)=-j(1/62.83)\approx-j0.0159$ S. In polar form: $0.0159\angle-90^\circ$ S.**
  - **Total Admittance (Ytotal): $Y_{total}=Y_R+Y_L=(0.02+j0)+(0-j0.0159)=0.02-j0.0159$ S.**
    - **In polar form: $|Y_{total}|=0.02^2+(-0.0159)^2$ $=0.0004+0.0002528$ $=0.0006528$ ≈0.02555 S.**
    - **$\theta=\arctan(-0.0159/0.02)=\arctan(-0.795)\approx-38.49^\circ$.**
    - **So, $Y_{total}=0.02555\angle-38.49^\circ$ S.**
  - **Total Current (Itotal): Assume source voltage is reference: $V_{source}=100\angle0^\circ$ V. $I_{total}=V_{source}\times Y_{total}=(100\angle0^\circ)\times(0.02555\angle-38.49^\circ)$ $I_{total}=(100\times0.02555)\angle(0^\circ-38.49^\circ)=2.555\angle-38.49^\circ$ A. (The total current lags the voltage, as expected for a predominantly inductive parallel circuit).**

5. Power in AC Circuits: Beyond Simple V×I

In DC circuits, power is straightforward (P=VI). However, in AC circuits, the presence of phase differences between voltage and current necessitates a more nuanced understanding of power, leading to concepts of real, reactive, and apparent power.

- **Instantaneous Power (p(t)):**
  - **Definition: The power at any given instant in time. It is the product of the instantaneous voltage and instantaneous current.**
  - **Formula: $p(t)=v(t)\times i(t)$**
  - **For a sinusoidal circuit, p(t) is also a sinusoidal waveform, but it oscillates at twice the supply frequency and typically has a non-zero average value.**
- **Average Power (Real Power) (P):**
  - **Definition: This is the actual power consumed by the resistive components of the circuit and converted into useful work (e.g., heat, mechanical energy). It is the average of the instantaneous power over one complete cycle. This is the power that does "real" work.**
  - **Units: Watts (W).**
  - **Formulas:**

        - $P = V_{RMS} I_{RMS} \cos\phi$ (where ϕ is the phase angle between the total voltage and total current).
        - $P = I_{RMS}^2 R_{total}$ (where R_{total} is the total equivalent resistance of the circuit).
        - $P = V_{R\_RMS}^2 / R$ (where V_{R_RMS} is the RMS voltage across the resistive part).
- **Reactive Power (Q):**
    - **Definition: This is the power that flows back and forth between the source and the reactive components (inductors and capacitors) of the circuit. It is absorbed during one part of the cycle and returned to the source during another. It does no net work but is essential for establishing and maintaining electric and magnetic fields.**
    - **Units: Volt-Ampere Reactive (VAR).**
    - **Formulas:**
        - $Q = V_{RMS} I_{RMS} \sin\phi$
        - $Q = I_{RMS}^2 X_{net}$ (where $X_{net} = X_L - X_C$ is the net reactance of the circuit).
        - $Q_L = I_{RMS}^2 X_L$ (positive VAR, for inductive components)
        - $Q_C = I_{RMS}^2 X_C$ (negative VAR, for capacitive components)
    - **By convention, reactive power associated with inductive loads is considered positive (lagging VARs), and reactive power associated with capacitive loads is considered negative (leading VARs).**
- **Apparent Power (S):**
    - **Definition: This is the total power that appears to be supplied by the source. It is the product of the total RMS voltage and total RMS current of the circuit, without considering the phase angle. It represents the total capacity of the power delivery system.**
    - **Units: Volt-Ampere (VA).**
    - **Formulas:**
        - $S = V_{RMS} I_{RMS}$
        - $S = \sqrt{P^2 + Q^2}$ (from the power triangle).
- **Power Factor (PF):**
    - **Definition: The ratio of the real power (P) to the apparent power (S). It indicates how effectively the apparent power is being converted into useful real power.**
    - **Formula:** $PF = \cos\phi = P/S$
    - **The power factor can range from 0 to 1.**
        - **PF=1 (unity): Occurs in purely resistive circuits or at resonance, where** $\phi = 0^\circ$**. All apparent power is real power.**
        - **PF<1: Indicates the presence of reactive components.**
    - **Lagging Power Factor: Occurs in inductive circuits, where current lags voltage (**$\phi > 0^\circ$**). Most industrial loads (motors, transformers) are inductive, leading to a lagging PF.**
    - **Leading Power Factor: Occurs in capacitive circuits, where current leads voltage (**$\phi < 0^\circ$**).**
- **Power Triangle:**

- Graphical Representation: The relationship between Real Power (P), Reactive Power (Q), and Apparent Power (S) can be visualized using a right-angled triangle, called the power triangle.
- Sides:
  - Hypotenuse: Apparent Power (S)
  - Adjacent Side: Real Power (P)
  - Opposite Side: Reactive Power (Q)
- The angle between P and S is the power factor angle ϕ.
- By Pythagorean theorem: S2=P2+Q2. This is a fundamental relationship in AC power.
- Significance: The power triangle helps in understanding the power dynamics in an AC circuit and is crucial for power factor correction (improving system efficiency by reducing reactive power).

- Numerical Example 5.1: An AC motor draws 5 kW of real power and 3 kVAR (inductive) of reactive power from a single-phase AC supply. Calculate the apparent power, total current drawn if the supply voltage is 230 V, and the power factor.
  - Real Power (P): P=5 kW =5000 W.
  - Reactive Power (Q): Q=3 kVAR =3000 VAR (inductive, so positive Q).
  - Apparent Power (S): Using the power triangle relationship: S=P2+Q2 =50002+30002 =25×106+9×106 =34×106 ≈5831 VA.
  - Total Current (IRMS): S=VRMSIRMS⟹IRMS=S/VRMS=5831/230≈25.35 A.
  - Power Factor (PF): PF=P/S=5000/5831≈0.857 lagging (since Q is positive/inductive).
    - The power factor angle ϕ=arccos(0.857)≈30.98∘.

6. Resonance in AC Circuits: Special Conditions

Resonance is a specific condition in an RLC circuit where the effects of inductance and capacitance cancel each other out, leading to unique characteristics.

- Definition of Resonance: Resonance occurs in an RLC circuit when the inductive reactance (XL) equals the capacitive reactance (XC). At this specific frequency (resonant frequency), the circuit's impedance becomes purely resistive, and the voltage and current are in phase, resulting in a unity power factor.
  - Condition for Resonance: XL=XC ωrL=1/(ωrC) ωr2=1/(LC) ωr=1/LC

- **Resonant Frequency (f_r): The frequency at which resonance occurs. Formula: $f_r=1/(2\pi LC$) (Hz)**
- **Series Resonance:**
    - **Circuit Configuration: Resistor, inductor, and capacitor are connected in series.**
    - **Impedance at Resonance: At f_r, $X_L=X_C$, so $Z_{total}=R+j(X_L-X_C)=R+j0=R$. This means the total impedance is purely resistive and at its minimum value.**
    - **Current at Resonance: Since impedance is minimum, the current in a series resonant circuit is maximum for a given applied voltage: $I_{max}=V_{source}/R$.**
    - **Voltage Magnification: Although the total impedance is just R, the individual voltages across the inductor ($V_L=I\times X_L$) and capacitor ($V_C=I\times X_C$) can be significantly larger than the applied source voltage, especially for high Q circuits. This is due to the phase opposition of V_L and V_C, which effectively cancel each other out.**
    - **Power Factor at Resonance: Power factor is unity (1), as $\phi=0^\circ$.**
    - **Applications: Resonant filters (band-pass filters), voltage amplifiers, radio receivers (tuning circuits).**
- **Parallel Resonance (Anti-resonance):**
    - **Circuit Configuration: Resistor, inductor, and capacitor are connected in parallel. Often, the resistor represents the inherent resistance of the inductor coil.**
    - **Admittance at Resonance: At f_r, $Y_{total}=G+j(B_C-B_L)$. When $X_L=X_C$, then $B_L=B_C$, so $Y_{total}=G=1/R$. This means the total admittance is purely conductive and at its minimum value.**
    - **Impedance at Resonance: Since admittance is minimum, the total impedance of a parallel resonant circuit is at its maximum value ($Z_{total}=R$).**
    - **Current at Resonance: Since impedance is maximum, the total current drawn from the supply is minimum for a given applied voltage.**
    - **Current Magnification: A large circulating current can flow between the parallel L and C components, even when the total current drawn from the source is minimal.**
    - **Power Factor at Resonance: Power factor is unity (1), as $\phi=0^\circ$.**
    - **Applications: Tank circuits in oscillators, band-stop filters, impedance matching circuits.**
- **Quality Factor (Q): The Sharpness of Resonance**
    - **Definition: A dimensionless parameter that quantifies the "sharpness" or selectivity of a resonant circuit. A higher Q factor means a sharper and narrower response curve (e.g., current vs. frequency for series resonance), indicating better energy storage relative to energy dissipation.**

  - For Series RLC Circuit (Q_s): Formula: $Q_s = X_L / R = \omega_r L / R = (1/R) L/C$ It also represents the voltage magnification at resonance (V_L / V_{source} or V_C / V_{source}).
  - For Parallel RLC Circuit (Q_p): (assuming resistor in parallel with LC branch) Formula: $Q_p = R / X_L = R / (\omega_r L) = RC/L$ It represents the current magnification in the tank circuit.
- Bandwidth (BW): The Range of Frequencies
  - Definition: The range of frequencies over which the power delivered to the circuit is at least half of the power delivered at resonance (half-power points). It's the difference between the upper and lower half-power frequencies ($f_2 - f_1$).
  - Formula: $BW = f_r / Q$
  - A high Q circuit has a narrow bandwidth (high selectivity), while a low Q circuit has a broad bandwidth.
- Numerical Example 6.1 (Series Resonance): A series RLC circuit has $R = 5\Omega$, $L = 100$ mH, and $C = 50\mu F$. Calculate its resonant frequency, quality factor, and bandwidth.
  - Resonant Frequency (f_r): $f_r = 1/(2\pi LC) = 1/(2\pi 0.1 \times 50 \times 10^{-6}) = 1/(2\pi 5 \times 10^{-6})$ $f_r = 1/(2\pi \times 0.002236) \approx 71.18$ Hz.
  - Inductive Reactance at Resonance (X_L): $X_L = 2\pi f_r L = 2\pi \times 71.18 \times 0.1 \approx 44.72$ Ω. (Note: X_C will also be 44.72Ω at f_r).
  - Quality Factor (Q_s): $Q_s = X_L / R = 44.72/5 = 8.944$.
  - Bandwidth (BW): $BW = f_r / Q_s = 71.18/8.944 \approx 7.96$ Hz. This means the circuit effectively responds to frequencies in a band of approximately 7.96 Hz around 71.18 Hz.

7. Three-Phase Balanced Circuits: Industrial Power

Three-phase AC power systems are the backbone of modern electrical grids, offering significant advantages over single-phase systems for generation, transmission, and heavy industrial applications.

- Advantages of Three-Phase Systems:
  - Efficient Power Transmission: For transmitting a given amount of power, a three-phase system requires less conductor material than an equivalent single-phase system, reducing transmission losses and costs.

- Constant Power Delivery: In a balanced three-phase system, the instantaneous total power delivered to the load is constant, unlike single-phase power which pulsates. This results in smoother torque production in motors and less vibration.
- Self-Starting Motors: Three-phase induction motors are inherently self-starting, producing a rotating magnetic field that eliminates the need for auxiliary starting windings or mechanisms often required in single-phase motors.
- Versatility: Can easily supply both three-phase loads (e.g., large industrial motors) and single-phase loads (e.g., lighting, domestic appliances) simultaneously.
- Higher Power Density: For a given frame size, three-phase generators and motors have a higher power output compared to single-phase machines.

- **Generation of Three-Phase Voltages:**
  - Three-phase voltages are generated by having three separate coils (windings) in a generator, mechanically displaced by 120° electrical degrees from each other. As the rotor (magnetic field) rotates, sinusoidal voltages are induced in each coil, with each voltage phase-shifted by 120° relative to the others.
  - If phase A voltage is $V_A=V_m\sin(\omega t)$, then phase B voltage is $V_B=V_m\sin(\omega t-120^\circ)$, and phase C voltage is $V_C=V_m\sin(\omega t-240^\circ)$ or $V_m\sin(\omega t+120^\circ)$.
- **Star (Wye) Connection (Y):**
  - Configuration: One end of each of the three phase windings (A, B, C) is connected to a common point, called the neutral point (N). The other three ends are brought out as the three line terminals (A, B, C).
  - Voltage Relations (Balanced System):
    - Phase Voltage (V_{ph}): Voltage measured between a line terminal and the neutral point (e.g., V_{AN}, V_{BN}, V_{CN}).
    - Line Voltage (V_L): Voltage measured between any two line terminals (e.g., V_{AB}, V_{BC}, V_{CA}).
    - Formula: $V_L=3\,V_{ph}$
    - The line voltages are 120° apart from each other, and they lead their respective phase voltages by 30°.
  - Current Relations (Balanced System):
    - Line Current (I_L): Current flowing in the line conductors.
    - Phase Current (I_{ph}): Current flowing through each phase winding or load connected to the phase.
    - Formula: $I_L=I_{ph}$
  - Neutral Current: In a perfectly balanced star-connected system, the sum of the three phase currents at the neutral point is zero ($I_A+I_B+I_C=0$). Thus, no current flows in the neutral wire. However, in an unbalanced system, a neutral current will flow.

- **Applications: Often used for transmission and distribution systems where a neutral wire is required to supply both three-phase and single-phase loads (e.g., household supply derived from one phase and neutral).**
- **Numerical Example 7.1 (Star Connection): A balanced star-connected load has a phase voltage of 230 V. Calculate the line voltage. If the phase current is 10 A, what is the line current?**
    - **Line Voltage (VL): $V_L=3 V_{ph}=3 \times 230 \approx 398.4$ V.**
    - **Line Current (IL): $I_L=I_{ph}=10$ A.**
- **Delta Connection (Δ):**
    - **Configuration: The three phase windings (or loads) are connected end-to-end to form a closed triangular loop. Each corner of the triangle forms a line terminal. There is no common neutral point.**
    - **Voltage Relations (Balanced System):**
        - **Formula: $V_L=V_{ph}$ (The voltage across each phase winding is directly the line-to-line voltage).**
    - **Current Relations (Balanced System):**
        - **Formula: $I_L=3 I_{ph}$**
        - **The line currents are 120° apart from each other, and they lag their respective phase currents by 30°.**
    - **Applications: Commonly used for high-power industrial loads (e.g., large motors) where a neutral connection is not required.**
- **Numerical Example 7.2 (Delta Connection): A balanced delta-connected load has a phase current of 15 A. What is the line current? If the line voltage is 400 V, what is the phase voltage?**
    - **Line Current (IL): $I_L=3 I_{ph}=3 \times 15 \approx 25.98$ A.**
    - **Phase Voltage (Vph): $V_{ph}=V_L=400$ V.**
- **Power in Three-Phase Circuits (Balanced Systems): The total power in a balanced three-phase system is simply three times the power in a single phase. The power factor $\cos\phi$ is the power factor of each phase.**
    - **Total Real Power (Ptotal):**
        - **Using Phase Quantities: $P_{total}=3V_{ph}I_{ph}\cos\phi$**
        - **Using Line Quantities: $P_{total}=3 V_L I_L \cos\phi$ (Note: The $\cos\phi$ here refers to the power factor angle of each phase load, i.e., the angle between phase voltage and phase current of that load).**

- **Total Reactive Power (Qtotal):**
  - **Using Phase Quantities: Qtotal=3VphIphsinϕ**
  - **Using Line Quantities: Qtotal=3 VLILsinϕ**
- **Total Apparent Power (Stotal):**
  - **Using Phase Quantities: Stotal=3VphIph**
  - **Using Line Quantities: Stotal=3 VLIL**
  - **Also, as with single-phase power: Stotal=Ptotal2+Qtotal2**
- **Power Factor (PF) of Three-Phase System:**
  - **PF=cosϕ=Ptotal/Stotal (same as for a single phase, assuming a balanced load).**

- **Numerical Example 7.3 (Three-Phase Power Calculation): A balanced three-phase star-connected load with a line voltage of 415 V draws a line current of 25 A at a power factor of 0.8 lagging. Calculate the total real power, total reactive power, and total apparent power drawn by the load.**
  - **Given: VL=415 V, IL=25 A, PF=cosϕ=0.8 (lagging).**
  - **Power Factor Angle (ϕ): ϕ=arccos(0.8)≈36.87∘.**
  - **Total Real Power (Ptotal): Ptotal=3 VLILcosϕ=3 ×415×25×0.8≈14378 W or 14.378 kW.**
  - **Total Apparent Power (Stotal): Stotal=3 VLIL=3 ×415×25≈17972 VA or 17.972 kVA.**
  - **Total Reactive Power (Qtotal): Qtotal=3 VLILsinϕ=3 ×415×25×sin(36.87∘) Qtotal=3 ×415×25×0.6≈10783 VAR or 10.783 kVAR (lagging).**

- **Verification: Ptotal2+Qtotal2 =143782+107832 ≈206733284+116273089 =323006373 ≈17972 VA. (Matches Stotal).**

Activities/Assessments:

To reinforce learning and assess understanding, the following activities and assessments are recommended:

- **Exercises on converting between time-domain and phasor representations:**
  - **Task 1: Given a time-domain voltage v(t)=200cos(120πt+75◦) V, convert it into its RMS phasor representation.**
  - **Task 2: Given an RMS current phasor I=15∠−150◦ A, and a frequency of 400 Hz, write its corresponding time-domain expression i(t).**
  - **Task 3: For two voltages v1(t)=50sin(ωt+30◦) V and v2(t)=70cos(ωt−45◦) V, express both in RMS phasor form and determine the phase difference between them (which one leads/lags by how much).**
- **Problem-solving for series/parallel AC circuits:**
  - **Task 1: A series RLC circuit consists of a 30Ω resistor, a 0.05 H inductor, and a 20μF capacitor. If it is connected to a 150 V, 60 Hz AC supply, calculate: a) Inductive reactance (XL) and Capacitive reactance (XC). b) Total complex impedance of the circuit (Ztotal). c) Total RMS current flowing through the circuit (Itotal). d) RMS voltage across each component (VR, VL, VC). e) Draw a simple phasor diagram showing Vsource, Itotal, VR, VL, and VC.**
  - **Task 2: A parallel circuit has a 100Ω resistor in one branch and a 0.08 H inductor in another branch. It is connected to a 240 V, 50 Hz AC supply. Calculate: a) Current flowing through the resistor branch (IR). b) Current flowing through the inductor branch (IL). c) Total current drawn from the supply (Itotal). d) Draw a simple phasor diagram showing Vsource, IR, IL, and Itotal.**
- **Calculations of power factor and power triangle components:**
  - **Task 1: An industrial load consumes 12 kW of real power and 9 kVAR of leading reactive power. a) Calculate the apparent power (S). b) Determine the power factor (PF) and state if it is leading or lagging. c) If the supply voltage is 400 V (single-phase), calculate the RMS current drawn by the load.**
  - **Task 2: A single-phase AC circuit has an RMS voltage of 230 V and draws an RMS current of 15 A. The current lags the voltage by 40◦.**

Calculate the real power, reactive power, apparent power, and power factor. Construct the power triangle for this circuit (conceptual sketch).

- **Simulation of resonant circuits (Conceptual/Software-based):**
    - **Task 1 (Series Resonance): Describe how you would set up a simulation to observe the effect of frequency on current in a series RLC circuit. Explain what you would expect to see on a graph of current versus frequency, highlighting the resonant frequency and explaining its significance.**
    - **Task 2 (Parallel Resonance): Describe how you would simulate a parallel RLC circuit and observe the effect of frequency on the total impedance (or total current drawn from the source). Explain the expected behavior and the significance of the resonant frequency in this context.**